**Electron – Electron Interaction**

Now let’s do a simple – he says – model of an insulator. Let’s consider the insulator to consist of atoms at positions **R**. We’ll consider an insulator held together via Van der Waals forces mainly. The unperturbed atomic eigenstates on each site we’ll consider to be obtained from a central field approximation, or some such, as discussed in the QM folder. We’ll label these states φν(**r**-**R**j), with energies εν. Let ajν, a†jν annihilate/create such states. Then we could say, using the tight-binding model and including only different-site Coulomb interactions, say, that our H is (see 2nd quantization file):



where,



This equation by itself is not exactly solvable of course. But we can make an approximation to turn it into a quadratic H, which is. The first approximation is to consider the excitations of each onsite atom, ωm = εjν´ - εjν, and introduce annihilation/creation operators, bjm, bjm†, that destroy/create these excitations. So there is a one to one correspondance:



We may imagine these b’s commute with each other [see bosonization file for a typical calculation, but I’m not going to do it], as excitations do, as long as there aren’t too many of them, as of course we cannot do the same excitation twice. This being so, we may write the first part of H as:



Then in regards the interaction, presuming the electrons are not likely to move much from their host atom, we can approximate the interaction as basically of a Harmonic oscillator type. Recall from the QM folder, when we did the Lattice H we found the interaction between two electrons on different sites **R**, **R**´ would be:



and further, we could say,



Then let’s fill that into our 2nd quantized interaction term,



Then for the whole PE term we’d have:



We can write this as:



(it’s the sum because a†iνaiν´´´ = b†im, say, when when ν has energy greater than ν´´´ but bkm for the other case) So altogether we have:



And we’ll note that this H is diagonalizable since it is only quadratic in the b operators. We may define the polarization operator,



**X**(m) is a vector which depends on the particular excitation, and is different, presumably, for every m. In terms of this we can write our H as:



Now as we usually do, we’d go to Fourier space. Define,



where k is restricted to BZ for uniqueness. And then we have:



where,



Can see that **P**(-**k**) = **P**†(**k**). As usual, it’s good to go to the basis which is diagonalizes **K**(k), which Mahan asserts in the small k limit comes to (for a cubic crystal):



The eigenvectors are the usual. One points along **k**, as can verify:



and two others would be ones perpendicular to **k**. These would have eigenvalues -1/3. So altogether we could say:



Let’s also say **ε**λ(-**k**) = -**ε**λ(**k**). Then,



Then we’d have:



So finally have, ignoring the irrelevant constant,



**Feynman Diagram Rules**

In this form, we can adapt the rules from the electron-phonon interaction. So let’s say we’re interested in calculating the (complex time) single particle GF for the electrons. So we wish to calculate, generally (recall the b’s are bosonic operators),



and of course these are given by:



where,



and,



Now let’s get the unperturbed phonon GF. This is:



We note



where,



Remember that boson number is not conserved and so there is no chemical potential. We can determine the complex time GF from,



Now note,



So,



And then,



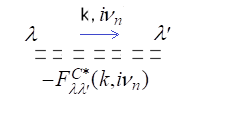
Aside from the e’s, ε’s and X’s, this is the same as phonon GF stuff. So taking the FT we will come again to:



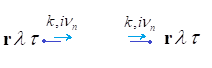
where the 2 is understood as outer product. The Feynman rules for single particle GF’s are, basically without any justification…

**Fourier Space Rules**

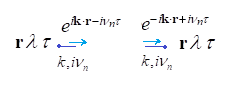
So we have our single GF’s:



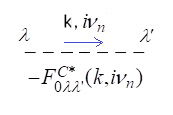
(now since the perturbation is diagonal in the indices, so will the full GF itself) and the external points:



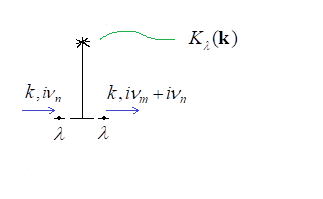
These carry the Fourier transform variable. And if we’re ultimately interested in the real space, time GF, then we must add the following factors to each:



and we connect these with the bare GF’s



The Vee interaction would be:



The frequency line is due to fact that Fourier transform of product introduces an extra frequency line – see that example in the Fourier transform file. Also, it might seem that momentum shouldn’t be conserved at the vertex, given the P(-q)P(q) form of the interaction term, but recall that the GF is of form PqPq† [and Pq† = P-q], and so to construct a GF, the P(-q) will have to convert to Pq†. And so then basically, the momentum flowing in will match the momentum flowing out. Same argument applies if we attach to the Pq first. Also, I’ve multiplied the vertex prefactor by two, because there are two ways to connect the vertex to a stray boson line.

**Topology**

Connect all topologically distinct, fully connected (meaning no vacuum bubbles), diagrams together, associating with each element in the diagram the indicated term. Remember that all energy-momentum labels **k**, ω must be going in the same way w/r to the GF arrow.

**Equal time issues**

None per se´

**Signs/Numerical Factors**

Diagrams get factor of one I believe.

**Sum**

Then sum/integrate over all independent momenta - wavenumbers/energies-frequencies/indices (and polarizations, spins).



Not sure about the (1/V)·Σk deal. Since the unperturbed GF’s are presumably diagonal in λλ´, the internal sum over polarizations will just cancel δ functions, with the net effect that whatever polarization we put it in will come out.